Presented at SDG Short Course II on Exploration and Development of Geothermal Resources,
organized by UNU-GTP, GDC and KenGen, at Lake Bogoria and Lake Naivasha, Kenya, Nov. 9-29, 2017.

RESISTIVITY METHODS – MT

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ABSTRACT

Magnetotellurics (MT) is a passive electrical resistivity method that probes the subsurface resistivity structure from a depth of a few tens of meters down to a depth of several tens or even hundreds of km. It is the only method that is capable of revealing the resistivitity at such a great depth. It is a passive method, using the magnetic variations of the geomagnetic field as its source. Contrary to active resistivity methods like, the geoelectrical methods where an injected current into the earth is the source and the TEM method where the source is a current loop.

The basic knowledge of the MT method includes acquisition, processing and interpretation. There exits a variety of books and articles on the method which has developed greatly the last decades as the dynamical range in data recording has inceased and computer capacities as well. A book that covers the practical aspects of applying the magntotelluric technique should recommeded: Practical Magnetotellurics by Simpson and Bahr (2005). Making use of the internet is always a good idea. Take a look at the webside, MTnet which is an internet forum for the free exchange of programs, data and ideas, especially for scientist in non-commercial studies. The address for the web page is: www.mtnet.info.

1. INTRODUCTION

The magnetotelluric (MT) method is a passive surface measurement of the earth‘s natural electrical (**E**) field and magnetic (**H**) field in orthogonal directions. It can be shown that the relationship between the horizontal orthogonal magnetic and electric fields depend on the subsurface resistivity structure. It is therefore used to determine the conductivity of the earth, ranging from a few tens of meters to several hundreds of kilometers. The fundamental theory was first developed by Cagniard (1950) and Tikhonov (1950, reprinted 1986). MT generally refers to recording of 10 kHz to 1000 s (0.001 Hz) or as low as 10.000 s (0.0001 Hz). AMT (audio MT) refers to „audio„ frequencies, generally recording > 100 Hz to 10 kHz. LMT-Long period MT generally refers to recording from 1.000 s to 10.000 s or much higher (to 100.000). The depth of penetration of MT soundings depends on the frequency, the lower the frequency the greater the depth of penetration and vice versa.

The earth‘s electromagnetic field contains a wide spectrum (Figure 1). The low frequencies are generated by ionospheric and magnetospheric currents caused by solar wind (plasma) interfering with the earth‘s magnetic field. Higher frequencies (< 1 Hz) are due to thunderstorms near equator distributed as guided waves between the earth and the ionosphere. The time varying magnetic field induces electric field and hence currents in the ground. By measuring variations in the magnetic and electric fields in the surface of the ground, information on the subsurface resistivity can be obtained.

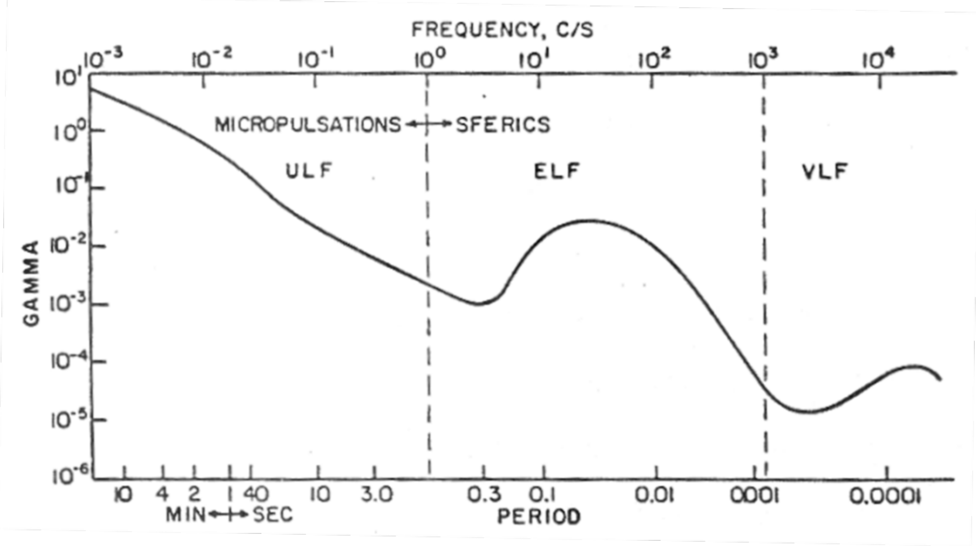

FIGURE 1: The natural magnetic field spectrum

The sunspot cycle has been measured since the mid 17th century. There is a regular cycle of little over 10 years. Presently there is a minimum of activity (see Figure 2), making MT soundings difficult in present years.

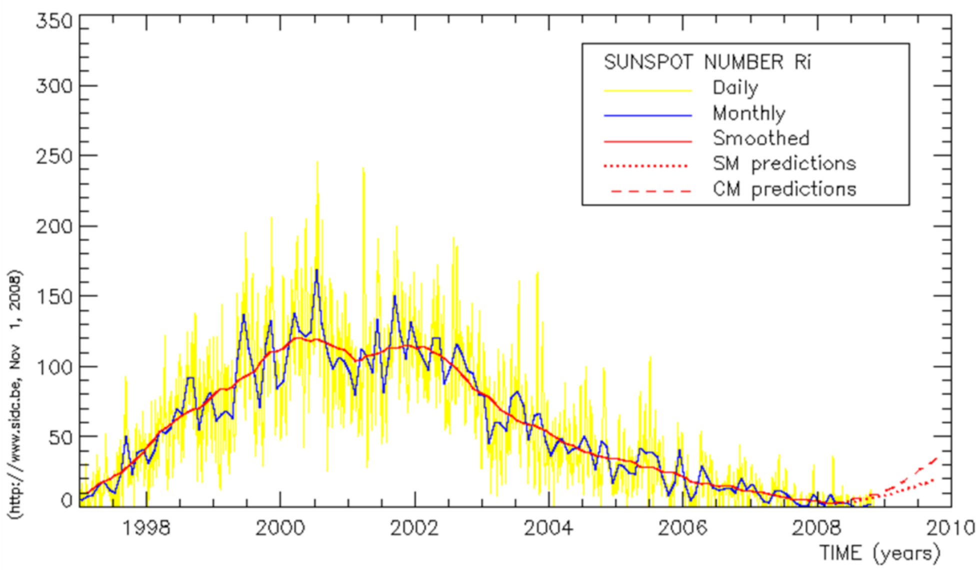

FIGURE 2: Current sunspot cycle

A typical setup for an MT sounding is shown in Figure 3. The horizontal orthogonal magnetic field, H_x (usually alligned in the magnetic north-south direction) and H_y (perpendicular to H_x) and the vertical magnetic field H_z are measured by magnetic coils. The horizontal orthogonal electric field, E_x and E_y are measured by a pair of electrodes (the potentional difference devided by the distance, 50-100 m). A GPS unit is used for syncronizing the data. The digital recording of the electromagnetic fields as a function of time is done through an aquisition unit and the time series saved on a memory card. The time series are Fourier transformed from the time domain to the frequency domain and processed. The tensors and the consequent apparent resistivity and phase are calculated as a funcion of the frequencies for later to be interpreted into a resistivity model of the subsurface (see Figure 5).

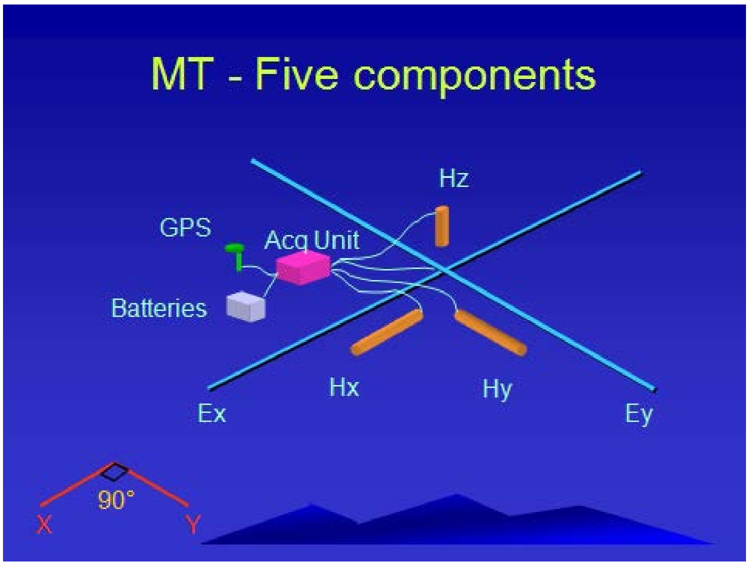

FIGURE 3: The setup of a magnetotelluric sounding

2. GENERAL THEORY OF ELECTROMAGNETISM

The four Maxwells's equations describe the electromagnetic field. They are Gauss's law for the magnetic field (div**B**=0) and Gauss's law for the electric field (div**D**=q), Faradays's law and Ampère's law with Maxwell's term:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \text{Faraday's law}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \qquad \text{Ampere's law}$$

E: Electrical field intensity (V/m)
H: Magnetic field intensity (A/m)
J: Electrical current density; **J=σE**
σ: conductivity (Simens/m); ρ=1/ σ (Ωm)
ε : electrical permittivity
μ: magnetic permeability

Assuming:

- Harmonic dependence of the oscillating electromagnetic fields; **H**,**E** ~ $e^{i\omega t}$;
  ω=2πν (angular frequency), $i=\sqrt{-1}$
  ν is frequency; ν=1/T; T is period;
  gives : $\frac{\partial \mathbf{H}}{\partial t} = i\omega \mathbf{H}; \quad \frac{\partial \mathbf{E}}{\partial t} = i\omega \mathbf{E}$
- Vertically incident plane wave
  i.e. $\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$

Using Faraday's law above, gives:

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu\frac{\partial H_x}{\partial t} \quad \Rightarrow \; -\frac{\partial E_y}{\partial z} = -\mu i\omega H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu\frac{\partial H_y}{\partial t} \quad \Rightarrow \; \frac{\partial E_x}{\partial z} = -\mu i\omega H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu\frac{\partial H_z}{\partial t} = 0$$

Similarily applying Ampère's law above, gives:

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \sigma E_x + \varepsilon\frac{\partial E_x}{\partial t} \quad \Rightarrow \; -\frac{\partial H_y}{\partial z} = (\sigma + i\omega\varepsilon)E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \sigma E_y + \varepsilon\frac{\partial E_y}{\partial t} \quad \Rightarrow \; \frac{\partial H_x}{\partial z} = (\sigma + i\omega\varepsilon)E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \sigma E_z + \varepsilon\frac{\partial E_z}{\partial t} = 0$$

Combining Faraday's and Ampère's law gives:

$$k^2 = i\omega\mu(\sigma + i\omega\varepsilon)$$

$$\sigma = 1/\rho; \quad \text{and} \quad \rho \approx 1 - 10^4 \;\; \Omega\text{m}, \;\; \text{or} \; \sigma \approx 1 - 10^{-4}\,\text{S/m}$$

$$\omega = 2\pi/T \;\; \text{and} \; T \approx 10^{-4} - 10^4 \; \text{seconds}$$

$$\varepsilon = \varepsilon_0\chi_e; \quad \varepsilon_0 = 8.85\cdot 10^{-12}; \; \text{and} \;\; \chi_e \approx 1 - 100$$

$$\Rightarrow \quad (\omega\varepsilon)_{max} = 2\pi\cdot 10^4\cdot 8.85\cdot 10^{-12}\cdot 100 \approx 5\cdot 10^{-5}$$

$$\Rightarrow \quad \sigma >> \omega\varepsilon$$

$$\Rightarrow \quad k^2 \approx i\omega\mu\sigma$$

For the quasi-stationary approximation:

$$\frac{\partial^2 E_y}{\partial^2 z} = i\omega\mu\frac{\partial H_x}{\partial z} = i\omega\mu(\sigma + i\omega\varepsilon)E_y$$

and:

$$\frac{\partial^2 E_x}{\partial^2 z} = -i\omega\mu\frac{\partial H_y}{\partial z} = i\omega\mu(\sigma + i\omega\varepsilon)E_x$$

Which can be written:

$$\frac{\partial^2 E_x}{\partial^2 z} = k^2 E_x \;; \qquad k^2 = i\omega\mu(\sigma + i\omega\varepsilon)$$

$$\frac{\partial^2 E_y}{\partial^2 z} = k^2 E_y$$

The general solutions for a homogeneous earth (σ constant) can be written as:

$$E_{x,y} = (A_{x,y}e^{kz} + B_{x,y}e^{-kz})e^{i\omega t}$$

$$H_x = \frac{1}{i\omega\mu}\frac{\partial E_y}{\partial z} = \frac{k}{i\omega\mu}(A_y e^{kz} - B_y e^{-kz})\,e^{i\omega t}$$

$$H_y = \frac{-1}{i\omega\mu}\frac{\partial E_x}{\partial z} = \frac{-k}{i\omega\mu}(A_x e^{kz} - B_x e^{-kz})\,e^{i\omega t}$$

$A_{x,y}$ and $B_{x,y}$ are constants to be determined

As $z\to\infty \Rightarrow H\to 0$ and $E\to 0 \Rightarrow A_{x,y}=0$; and we get:

$$\mathbf{E}_x = B_x e^{-kz}e^{i\omega t} \quad ; \quad \mathbf{E}_y = B_y e^{-kz}e^{i\omega t}$$

$$\mathbf{H}_x = \frac{-k}{i\omega\mu}B_y e^{-kz}e^{i\omega t} = \frac{-k}{i\omega\mu}\mathbf{E}_y$$

$$\mathbf{H}_y = \frac{k}{i\omega\mu}B_x e^{-kz}e^{i\omega t} = \frac{-k}{i\omega\mu}\mathbf{E}_x$$

The impedance tensor elements Z_{ij} are defined as:

$$Z_{xy} = \frac{E_x}{H_y} = \frac{i\omega\mu}{k} \approx \frac{i\omega\mu}{\sqrt{i\omega\sigma}} = \sqrt{\omega\mu\rho}\cdot e^{i\pi/4}$$

$$Z_{yx} = \frac{E_y}{H_x} = \frac{-i\omega\mu}{k} = -Z_{xy}$$

and we can calculate the resistivity of the half -space

$$\rho = \frac{1}{\omega\mu}\left|Z_{xy}\right|^2 = \frac{1}{\omega\mu}\left|Z_{yx}\right|^2$$

The resistivity may also we written as:

$$\rho = \frac{1}{\omega\mu}\left|\frac{E}{H}\right|^2 = \frac{T}{2\pi\mu}\left|\frac{E'\cdot 10^{-6}\cdot\mu}{B'\cdot 10^{-9}}\right|^2 = \frac{T\mu}{2\pi}\left|\frac{E'}{H'}\right|^2\cdot 10^6 = 0.2T\left|\frac{E'}{H'}\right|^2$$

For a none homogeneous earth the apparent resistivity (ρ_a) and phase (Θ_a) are defined as.

$$\rho_a = \frac{1}{\omega\mu}\left|Z_0\right|^2 \quad ; \quad \theta_a = \arg(Z_0) \neq 45^\circ$$

Z_0 is impedence at surface

The general definision of the impedance tensor is:

$$\vec{\mathbf{E}} = \begin{bmatrix}\mathbf{E}_x\\ \mathbf{E}_y\end{bmatrix} = \begin{bmatrix}\mathbf{Z}_{xx} & \mathbf{Z}_{xy}\\ \mathbf{Z}_{yx} & \mathbf{Z}_{yy}\end{bmatrix}\begin{bmatrix}\mathbf{H}_x\\ \mathbf{H}_y\end{bmatrix} = \overline{\overline{\mathbf{Z}}}\vec{\mathbf{H}}$$

For 1D earth, $Z_{xx} = Z_{yy} = 0$ and $Z_{xy} = -Z_{yx}$.
For 2D earth is possible to rotate the tensor in the strike direction and $Z_{xx} = Z_{yy} = 0$.
For 3D earth it is not possible to rotate the tensor and get $Z_{xx} = Z_{yy} = 0$.

Figure 4 shows the apparent resistivity and phase as a function of the period for an MT station. The two resistivities ρ_{xy} and ρ_{yx} are not equal. The resistivity structure is therefore not 1D. The strike direction (Zstrike) is around 20°, giving an indication of a 2D structure, since Zstrike is more or less the same value for all frequencies and the skew is low. The coherency is close to one, which means that there is a good correlation between the electric and magnetic fields.

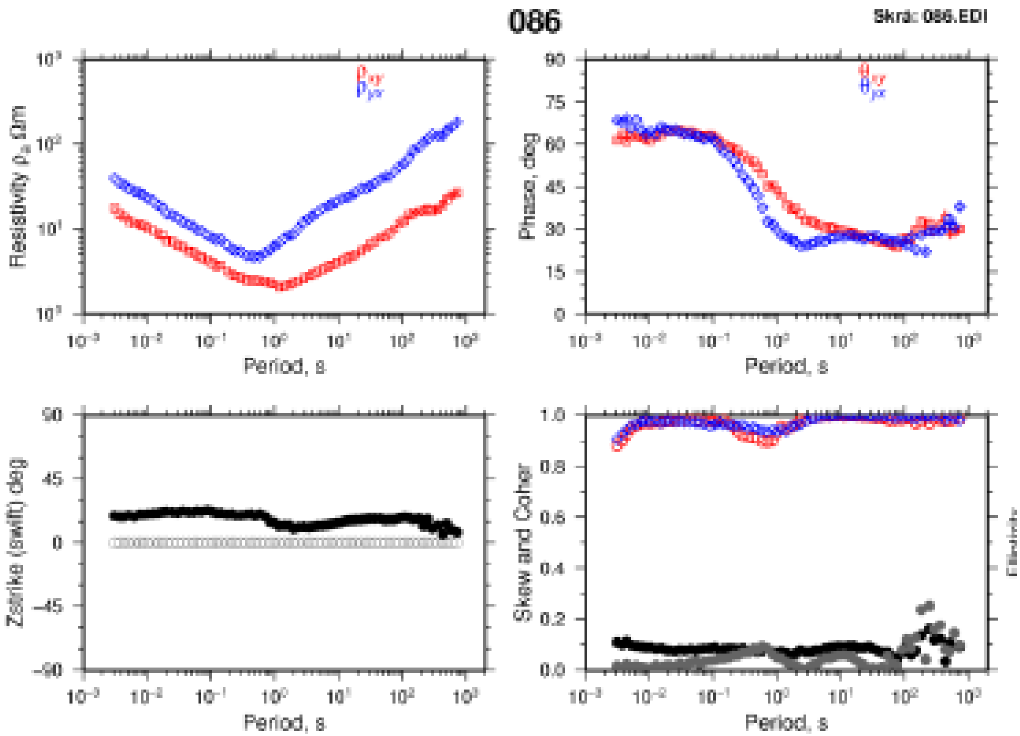

FIGURE 4: Apparent resistivity and phase, Zstrike, Skew and coherency for an MT sounding (taken from Hersir et al., 2009)

Figure 5 shows the joint inversion of the MT sounding and a TEM sounding measured at the same location.

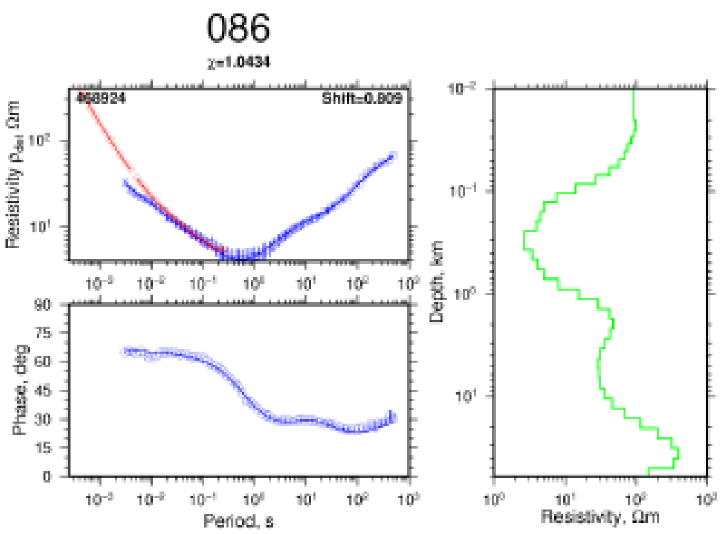

FIGURE 5: Joint inversion of TEM MT data. Red dots denote TEM data and the blue ones MT data. The resistivity model is shown in green to the left (taken from Hersir et al., 2009)

REFERENCES

Cagniard, L., 1953. Basic theory of the magnetotelluric method of geophysical prospecting. *Geophysics, 18,* 605-645.

Hersir, G.P., Rosenkjær, G.K., Vilhjálmsson, A.M., Eysteinsson, H., and Karlsdóttir, R., 2009: *The Krýsuvík geothermal field. Resistivity soundings 2007 and 2008* (in Icelandic). Iceland GeoSurvey - ISOR, Reykjavik, report ISOR 2009/047.

Simpson, F., and Bahr, K., 2005: *Practical magnetotellurics.* Cambridge University Press, Cambridge, 270 pp.

Tikhonov, A.N., 1986: On determining electrical characteristics of the deep layers of the Earth's crust. *Dokl. Akad. Nauk. USSR, 73,* 295-297.