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GEOTHERMAL DIVISION

Correcting for the Coastal Effect on the Apparent Resistivity of Schlumberger Soundings

Gylfi Páll Hersir

OS-88019/JHD-10 B May 1988

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CONTENTS

1. Introduction

It is well known that the apparent resistivity of a Schlumberger sounding carried out near the sea-shore is influenced by the ocean. Since sea-water has resistivity of about 0.2 Ωm and the resistivity of the rocks is typically 2-3 orders of magnitudes higher, the current tends to flow within the almost perfectly conducting sea, causing reduced potential and hence lower measured apparent resistivity.

Mundry and Worzyk (1979), quoting a formula from Sommerfeld (1897) which later was corrected by Carslaw (1899), calculated the apparent resistivity from the gradient of a potential, parallel and perpendicular to the edge of an infinitely thin and perfectly conducting sheet covering a half-plane on the surface of a homogeneous earth. (There is, however, an error in their results for a measurement parallel to the edge. The denominator in the parenthesis in equation (10) in their article should read $1+u^2$, instead of $1+u$).

In this report we will show how an infinitely thin and perfectly conducting sheet covering a half-plane on the surface of a homogeneous half-space (i.e. the ocean) influences the apparent resistivity values of a Schlumberger sounding. The effect of the ocean is calculated for an arbitrary distance of the sounding centre from the sheet and for an arbitrary orientation of the transmitting dipole. A computer program (written in FORTRAN) has been developed which calculates the influence and performs the correction for the coastal-effect. The input is the measured apparent resistivity values, the perpendicular distance form the centre of the Schlumberger sounding to the sea and the angle that the transmitting dipole makes with the coastal-line. Examples of application are given.

2. Theory

Let us consider a uniform half-space with resistivity, ρ_1 and an infinitely thin and perfectly conducting sheet covering a half-plane, see figure 1. We introduce a Cartesian coordinate system such that the xy-plane coincides with the surface of the half-space and the conducting sheet covers the $x \geq 0$ part of the xy-plane. It has been shown (Sommerfeld 1897, Carslaw 1899, Mundry and Worzyk 1979) that the potential V(x,y) on the surface of the half-space due to a point current source $I(x_1, y_1)$ also on the surface, where x and $x_1 \leq 0$ (see figure 1), is given by:

$$V = \frac{I\rho_1}{\pi^2}\frac{1}{R}\arctan(u_1)\,, \tag{1}$$

where

$$u_1 = \frac{2(xx_1)^{1/2}}{R}\,, \tag{2}$$

$$R = \left[(x - x_1)^2 + (y - y_1)^2\right]^{1/2} \tag{3}$$

and ρ_1 is the resistivity of the half-space $z<0$.

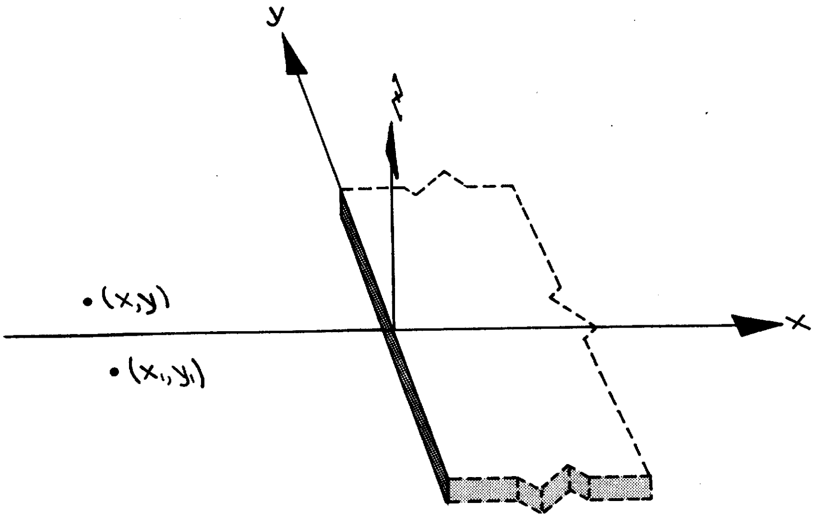

Figure 1: An infinitely thin and perfectly conducting sheet covering a half-plane coinciding with the half-plane $z=0$, $x\geq 0$ in Cartesian coordinates

In the particular case when the observation point (x,y) approaches the current source at (x_1, y_1), R decreases, u_1 increases and $\arctan(u_1)$ approaches $\pi/2$ and therefore equation (1) becomes:

$$\lim_{R\to 0} V = V_0 = \frac{I\rho_1}{2\pi}\frac{1}{R}, \tag{4}$$

which is the potential for a uniform half-space without the thin sheet.

The apparent resistivity (ρ_a) in a Schlumberger sounding is proportional to the voltage difference (ΔV) between two points (the receiving dipole) i.e. $\rho_a \propto \Delta V$. Since the receiving dipole is usually small compared to the transmitting dipole, the voltage difference can be approximated by the voltage gradient, at the sounding centre and along the direction of the transmitting dipole, times the length of the receiving dipole (l) i.e. $\Delta V = \partial V/\partial s \cdot l$, where the directional derivative is given by:

$$\frac{\partial V}{\partial s} = (\nabla V)\cdot s = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}\right)\cdot(\cos\psi, \sin\psi) = \frac{\partial V}{\partial x}\cos\psi + \frac{\partial V}{\partial y}\sin\psi, \tag{6}$$

and s is a unit vector along the dipole making the angle, ψ with the x-axis, see figure 2. Thus, $\rho_a \propto \partial V/\partial s$ and therefore we can write the ratio between the apparent resistivity values in the thin sheet model and the homogeneous model, ρ_a and ρ_1 respectively as:

$$\rho_a = \rho_1 \left\{ \frac{\partial V/\partial s}{\partial V_0/\partial s} \right\}. \tag{5}$$

Note that the apparent resistivity of a homogeneous model is the same as the resistivity of the homogeneous half-space.

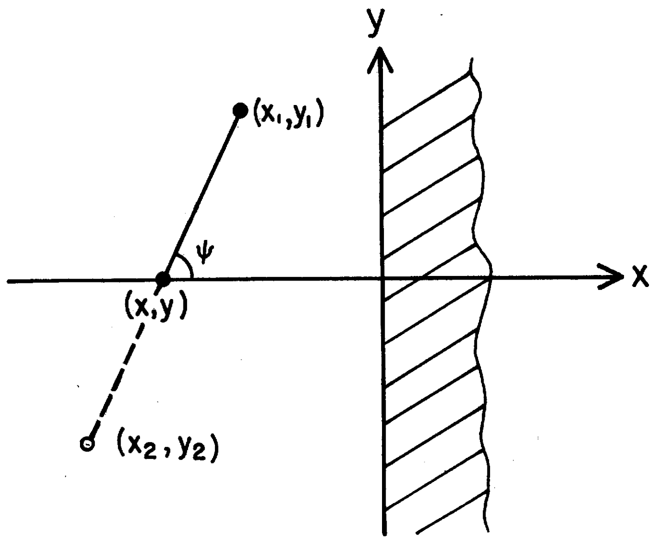

Figure 2: A point current source at (x_1, y_1) and (x_2, y_2)

Let us now consider one current source at the point (x_1, y_1) and calculate the electric field, $\partial V/\partial s$, at a point (x,y) and along the line to the current source, see figure 2. Note that the receiving dipole and the current source are on the same line, parallel to the unit vector s.

First we differentiate equation (1) with respect to x.

$$\frac{\partial V}{\partial x} = \frac{I\rho_1}{\pi^2} \frac{R\frac{\partial \arctan(u_1)}{\partial x} - \frac{\partial R}{\partial x}\arctan(u_1)}{R^2}, \tag{7}$$

where

$$\frac{\partial R}{\partial x} = \frac{x - x_1}{R}, \tag{8}$$

and

$$\frac{\partial \arctan(u_1)}{\partial x} = \frac{1}{1+u_1^2}\frac{\partial u_1}{\partial x} = \frac{R^2 x_1 - 2(xx_1)(x - x_1)}{(1+u_1^2)R^3(xx_1)^{1/2}}. \tag{9}$$

Equation (7) therefore becomes:

$$\frac{\partial V}{\partial x} = \frac{I\rho_1}{\pi^2}\frac{\dfrac{R^2 x_1 - 2(xx_1)(x - x_1)}{(1+u_1^2)R^2(xx_1)^{1/2}} - \dfrac{x - x_1}{R}\arctan(u_1)}{R^2}$$

$$= \frac{I\rho_1}{\pi^2}\frac{\dfrac{2x_1 - u_1^2(x - x_1)}{(1+u_1^2)Ru_1} - \dfrac{x - x_1}{R}\arctan(u_1)}{R^2}. \tag{10}$$

Similarly we find for the partial derivatives with respect to y

$$\frac{\partial R}{\partial y} = \frac{y - y_1}{R}, \tag{11}$$

and

$$\frac{\partial \arctan(u_1)}{\partial y} = \frac{1}{1+u_1^2}\frac{\partial u_1}{\partial y} = -\frac{2(xx_1)^{1/2}}{1+u_1^2}\frac{y - y_1}{R^3} = -\frac{u_1}{1+u_1^2}\frac{y - y_1}{R^2}. \tag{12}$$

Thus the partial derivative of the potential with respect to y is given by:

$$\frac{\partial V}{\partial y} = -\frac{I\rho_1}{\pi^2}\frac{\dfrac{u_1}{1+u_1^2}\dfrac{y - y_1}{R} + \dfrac{y - y_1}{R}\arctan(u_1)}{R^2}. \tag{13}$$

The directional derivative of the potential for the homogeneous half-space is:

$$\frac{\partial V_0}{\partial s} = -\frac{I\rho_1}{2\pi R^2}\left[\frac{x - x_1}{R}\cos\psi + \frac{y - y_1}{R}\sin\psi\right], \tag{14}$$

and according to figure 2, we note that

$$\cos\psi = \frac{x_1 - x}{R}, \quad \sin\psi = \frac{y_1 - y}{R}, \tag{15}$$

and since, $\sin^2(\psi) + \cos^2(\psi) = 1$, equation (14) becomes:

$$\frac{\partial V_0}{\partial s} = \frac{I\rho_1}{2\pi R^2}. \tag{16}$$

Now we can calculate the apparent resistivity, ρ_{a_1} that would be measured at (x,y) due to a current source monopole at (x_1, y_1). By using equations (6), (10), (13), (15) and (16) together with (5) we get:

$$\rho_{a_1} = \frac{2\rho_1}{\pi}\left[\frac{2x_1}{(1 + u_1^2)Ru_1} - \frac{u_1}{1 + u_1^2}\frac{x - x_1}{R} - \frac{x - x_1}{R}\arctan(u_1)\right]\cos\psi$$

$$- \frac{2\rho_1}{\pi}\left[\frac{u_1}{1 + u_1^2}\frac{y - y_1}{R} + \frac{y - y_1}{R}\arctan(u_1)\right]\sin\psi$$

$$= \frac{2\rho_1}{\pi}\left[\arctan(u_1) + \frac{u_1}{1+u_1^2} + \frac{2x_1\cos\psi}{(1+u_1^2)Ru_1}\right]. \tag{17}$$

In order to get the disturbed apparent resistivity, ρ_{a_2} measured at (x,y) due to a current source monopole at (x_2, y_2), where (x_2, y_2) is a point reflection image of (x_1, y_1) with respect to (x,y), we simply replace x_1 by x_2, y_1 by y_2 and ψ by $\psi+\pi$ (see figure 2) and find:

$$\rho_{a_2} = \frac{2\rho_1}{\pi}\left[\arctan(u_2) + \frac{u_2}{1+u_2^2} - \frac{2x_2\cos\psi}{(1+u_2^2)Ru_2}\right], \tag{18}$$

where

$$u_2 = \frac{2(xx_2)^{1/2}}{R}. \tag{19}$$

3. Application

In Schlumberger soundings there are two point current sources $+I(x_1, y_1)$ and $-I(x_2, y_2)$. The apparent resistivity is a superposition of the two apparent resistivities, ρ_{a_1} and ρ_{a_2}, from the two point current sources, as calculated in equations (17) and (18). Therefore:

$$\rho_a = \frac{\rho_{a_1} + \rho_{a_2}}{2}$$

$$= \frac{\rho_1}{\pi}\left[\arctan(u_1) + \arctan(u_2) + \frac{u_1}{1+u_1^2} + \frac{u_2}{1+u_2^2}\right]$$

$$+ \frac{2\rho_1\cos\psi}{R\pi}\left[\frac{x_1}{(1+u_1^2)u_1} - \frac{x_2}{(1+u_2^2)u_2}\right], \tag{20}$$

where R is half the distance between the current electrodes, x is the perpendicular distance from the coast to the centre of the sounding, ψ is the angle between the x-axis and the transmitting dipole,

$$u_1 = \frac{2(xx_1)^{1/2}}{R}, \quad u_2 = \frac{2(xx_2)^{1/2}}{R} \tag{21}$$

and

$$x_1 = R\cos\psi + x, \quad x_2 = x - R\cos\psi . \tag{22}$$

Now the true resistivity (ρ_1) of the half-space, can easily be calculated by equation (20), using the measured apparent resistivity (ρ_a), the perpendicular distance from the centre of the sounding to the coast (x) and the angle that the sounding makes with the coastal-line ($\phi = 90° - \psi$).

Equation (20) is of course exact only for a homogeneous half-space. In an earth having a non-homogeneous resistivity structure the resistivity contrasts within the earth are much less than the resistivity contrast between the earth and the sea. Therefore equation (20) can be applied as a first approximation in correcting for the coastal effect on the apparent resistivity of Schlumberger soundings.

A computer program has been written in FORTRAN for calculating equation (20). It is listed in the appendix. Four examples of its application for real data are shown in figure 3. The centres of the soundings are 100 m, 300 m, 500 m and 800 m from the sea. All the soundings have the transmitting dipole parallel to the coast.

The results show that the sea begins to affect the measured apparent resistivity when half the distance between the current electrodes is approximately the same as the distance from the centre of the sounding to the sea. It is also seen that the correction can be quiet drastic.

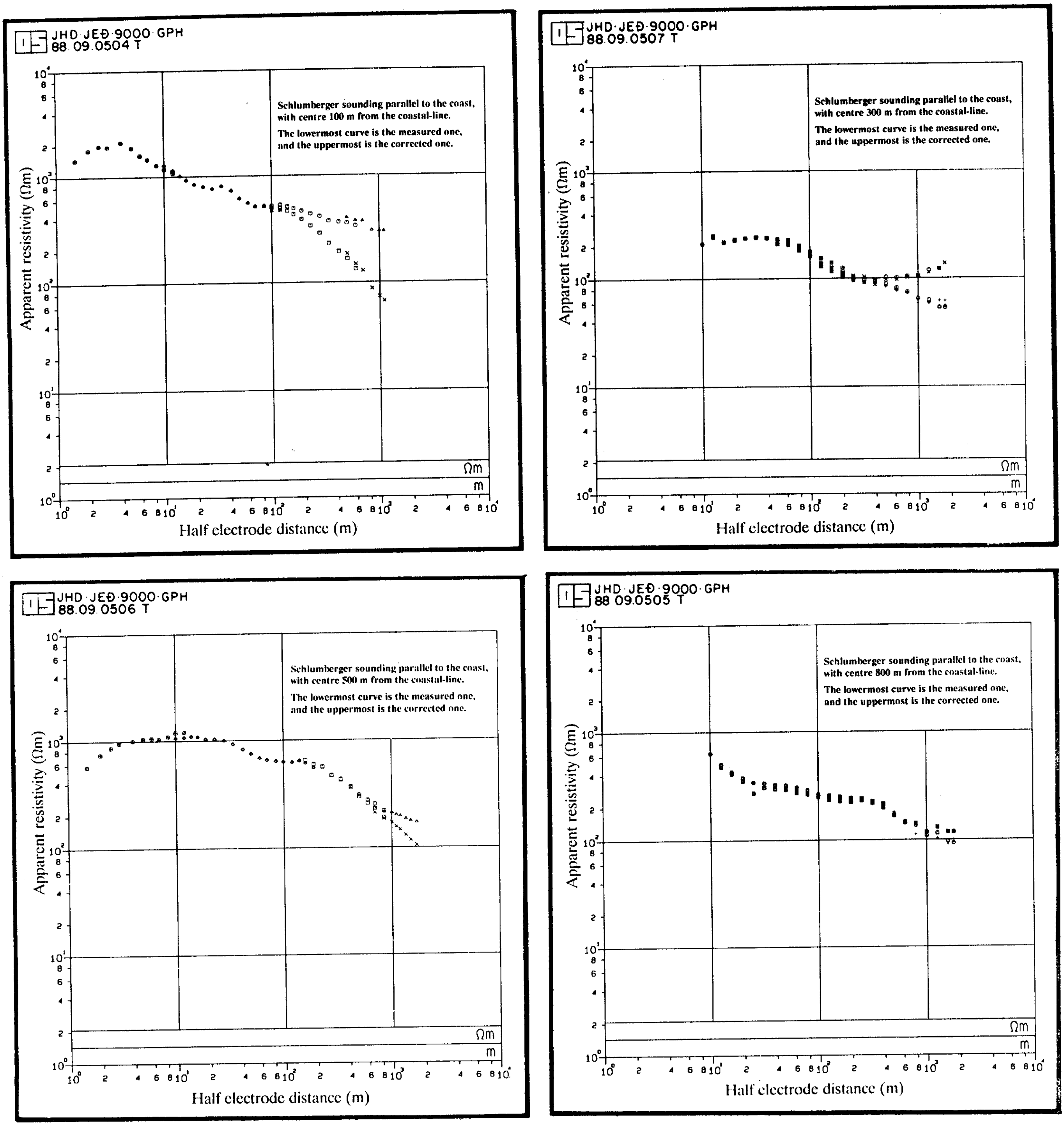

Figure 3: Examples of correction for the coastal effect on the apparent resistivity of Schlumberger soundings

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Hjálmar Eysteinsson and Knútur Árnason read the manuscript and made some critical comments.

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Mundry E. and P. Worzyk, 1979: On the coastal effect on geoelectrical soundings. Journal of Geophysics, 45, 329-336.

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Appendix

A FORTRAN-routine for correcting for the coastal effect of the apparent resistivity of Schlumberger soundings

```
C******************************************************************************
c
c                        SEACORR
c
c       THIS PROGRAM CORRECTS FOR THE COASTAL EFFECT ON THE APPARENT
c       RESISTIVITY OF SCHLUMBERGER SOUNDINGS.
c       THE MODEL CONSISTS OF AN INFINITELY THIN AND PERFECTLY CONDUCTING
c       SHEET COVERING A HALF-PLANE ON THE SURFACE OF A HOMOGENEOUS EARTH.
c
c       REFERENCE: GYLFI PÁLL HERSIR, 1988: CORRECTING FOR THE COASTAL
c       EFFECT ON THE APPARENT RESISTIVITY OF SCHLUMBERGER SOUNDINGS.
c       ORKUSTOFNUN OS-88019/JHD-10 B.
c
*******************************************************************************
c
        PROGRAM SEACORR
c
        CHARACTER*40 INPUT,OUTPUT
        CHARACTER*8 TEXTLINE
        LOGICAL EXIST
c
3       WRITE (*,'(a,$)') ' Input file: '
        READ (*,'(a)') INPUT
        INQUIRE(FILE=INPUT,EXIST=EXIST)
        IF(.NOT. EXIST) THEN
                PRINT *,CHAR(7),CHAR(7)
                PRINT *,'This file does not exist, try again'
                GO TO 3
        ENDIF
7       WRITE (*,'(a,$)') ' Output file: '
        READ (*,'(a)') OUTPUT
        WRITE (*,'(a,$)')
     &' Write the distance of the sounding from the sea (in meters):'
        READ (*,*) X
        WRITE (*,'(a,$)')
     &' Write the angle (in degrees) between the sea and the sounding:'
        READ (*,*) FIN
        FIN=90-FIN
c
        OPEN ( UNIT=1, FILE= INPUT, STATUS= 'OLD', ERR=3)
        OPEN ( UNIT=2, FILE= OUTPUT,  STATUS= 'UNKNOWN', ERR=7)
c
        CALL READHEAD !Writing the head in the output file
        DO 2 I=1,1000
                READ(1,*,ERR=901,END=100) NR,R,P,RHO,D
                RHOC=RCORR(R,RHO,X,FIN)
                WRITE(2,1) NR,R,P,RHOC,D
2       CONTINUE
1       FORMAT(T5,I2,T13,F7.2,T29,F5.1,T46,E10.4,T65,F4.1)
100     STOP
901     CLOSE (UNIT=1)
        CLOSE (UNIT=2)
        STOP 'ERROR READING DATA'
        END
C---------------------------------------------------------------------
        FUNCTION RCORR(R,RHO,XIN,FIN)
        PI=3.1415926
        FI=FIN*PI/180.
        X=-XIN
        XS1=R*COS(FI)+X
        XS2=X-R*COS(FI)
        U1=2.*SQRT(X*XS1)/R
        U2=2.*SQRT(X*XS2)/R
        RCORR1=ATAN(U1)+U1/(1.+U1*U1)+2.*XS1*COS(FI)/((1.+U1*U1)*R*U1)
        RCORR2=ATAN(U2)+U2/(1.+U2*U2)-2.*XS2*COS(FI)/((1.+U2*U2)*R*U2)
        RCORR= RHO*PI/(RCORR1+RCORR2)

        RETURN
        END
C---------------------------------------------------------------------
        SUBROUTINE READHEAD
        CHARACTER*80 LINE
        DO 3 I=1,16
                READ(1,'(A80)',ERR=901) LINE
                WRITE(2,'(A80)') LINE
3       CONTINUE
        RETURN
901     STOP 'ERROR READING IN READHEAD'
        END
```