THE HYDROLOGICAL SURVEY OF ICELAND
ADVISORY REPORT TO
STATE ELECTRICITY AUTHORITY

APPENDIX I
FLOOD ROUTING PROCEDURES

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APPENDIX I

FLOOD WAVE ROUTING

A. INTRODUCTION

Investigation and development of the hydroelectric resources of Iceland will require the study of a number of storage reservoirs to determine their best operation and their effect on flows in downstream sections of the rivers. This appendix presents some of the theory of water travel through reservoirs and in natural river channels under various conditions. It also presents accepted methods for routing flood waves through reservoirs and down natural river channels. These methods are used to determine the movement of flood waves and their alteration in shape as they move downstream.

B. FLOOD WAVE THEORY

Flood Waves

Runoff, resulting from precipitation, snow melt, or glacier melt, when it enters a stream, causes the stream to rise to a crest, and to recede as the runoff ceases. This rise, crest, and recession in stream flow is called a flood wave. A series of flood waves, varying in shape and size and occurring at irregular intervals make up a stream hydrograph. The rapid rise of a flood wave in a river channel produces unequal flow at different points and disturbs the usual balance between frictional resistance and energy gradient that exists during steady flow.

a. Propagation of Waves. A flood wave as it travels down a river channel is generally considered as a solitary wave of translation that follows the same general laws as a Monoclinal wave. It has both a rising and a falling face and a base that may be many miles long. There are broadly two general classes of flood waves, namely, those in which momentum or accelerative forces predominate, and those in which such forces have been counterbalanced by frictional resistance. Flood waves of the first class occur in power-regulated channels and in steep mountain channels as a result of cloudbursts or very high intensity rainfall. Most waves in natural river channels are of the second class, in which the time length of their base at any place greatly exceeds the time of travel from the headwater to the given place and accelerating forces are probably quite small.

b. Alteration of Wave Shape. Progressive flow and pondage, or reservoir action, both affect the movement of flood waves through natural river channels. Progressive flow consists of downstream movement of the flood wave without appreciable change in its shape. This is often referred to as flood wave translation, and is approximated only in prismatic river channels where stage and discharge relations remain approximately the same for the entire length of the channel. Wave velocity also remains nearly constant throughout the stages produced by the passage of the wave. The flood wave under these conditions may be considered as being translated in time essentially unchanged in shape and volume from one point to another in the channel. Reservoir, or pondage action, in the second process transforms the wave in both time and shape. It reduces the crest discharge, extends the period of maximum discharge, and lengthens the base of the wave.

Channel, or Valley Storage

Natural river channels usually consist of a succession of pools, or ponds and rapids or other control features, such as bends and channel constrictions. The water surfaces and the associated storage in the pools are regulated, or controlled to a variable extent, by the stage-discharge relationship of the control at their downstream end. Flow through rapids, bends, and channel constrictions, is controlled by the shape and the hydraulic characteristics of these channel sections.

When a flood wave enters a section of a river channel, the water surface must rise in order to maintain the increased flow. This rise in water surface elevation requires that a part of the water remain in temporary storage in the channel in order to maintain the increased stage. This temporary storage is called channel, or valley storage. Thus during the period when river stages are rising, a portion of the inflow is filling up this storage and reducing the water available for discharge at the lower end of the river section. After the peak of the flood wave passes and stages begin to fall, this temporary storage is released and helps to maintain stages and discharge at the lower end of the section.

Reservoir Storage

Reservoirs constructed in natural river channels act in much the same way as pools. However, they are designed so that both outflow and storage in the reservoir can be controlled. Thus storage in this case becomes a function of outflow alone, and the relationship between these two quantities is relatively easy to determine. The ideal reservoir may be defined as one whose depth is very great so that water velocity through it approaches zero. The surface of such a pool would be level and the displacement of a mass of water entering the upper end of the reservoir would be transmitted to all parts of the pool almost instantly. Many large, deep reservoirs approach this ideal condition and pool elevations at their lower ends reflect their storage within reasonably close limits. Short, shallow reservoirs differ radically from the ideal. Their pools are not level, but have a sloping profile, which varies with their inflow and the channel characteristics throughout much of their length. Their storage includes not only that below a level pool, but also that between the level pool and the backwater curves at their upper end. All of this storage must be considered when operating small, shallow reservoirs.

Storage Equation

A general hydrologic equation can be written for any natural section of a river or for a reservoir on a river if certain basic assumptions are made. These are, that accretions or losses from ground-water are negligible or equal; that no precipitation occurs over the channel, or reservoir, during the passage of the flood wave; and that evaporation losses be neglected. This equation reduces to:

$$S_1 + \int_{t_1}^{t_2} I dt = S_2 + \int_{t_1}^{t_2} O dt \quad . . . . (1)$$

where

S_1 is the storage in the section or reservoir at the beginning of a period.

S_2 is the storage at the end of the period.

I is the volume of inflow during the period from t_1 to t_2.

O is the volume of outflow during the period from t_1 to t_2.

Transposing Equation (1) becomes

$$\int_{t_1}^{t_2} I dt - \int_{t_1}^{t_2} O dt = S_2 - S \quad . . . . (2)$$

Equation (2) says simply that "The total quantity of inflow into a section of river or reservoir, during a given time period, minus the total quantity of outflow from the section during the same period is equal to the change in volume of water stored in the reach."

Neither I nor O can be expressed mathematically in terms of t, for any but the simplest cases, therefore arithmetic intergration becomes necessary for the solution of Equation (2). If a time interval t or Δt, be selected sufficiently short so that both I and O may be considered linear functions of t, during each interval, then Equation (2) may be rewritten in the following form:

$$(t_2 - t_1)\left(\frac{I_2 + I_1}{2} - \frac{O_2 + O_1}{2}\right) = S_2 - S_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

Equation (3) is generally called the Storage Equation.

C. FLOOD WAVE ROUTING

Definition

The process of determining progressively the timing and shape of a flood wave at successive points along a river, or through a reservoir is called Flood Routing.

Need for Flood Wave Routing

Development of the rivers of Iceland for hydroelectric power production, will require design and construction of both storage reservoirs and headwater ponds. Operation of these will materially affect downstream flows, particularly any flood waves moving down these streams. The flood routing procedure will be necessary to determine these downstream effects and must be included as a part of the investigations for hydroelectric power development. Investigation of flood wave movements have been carried out by many hydrologists during the past 60 years. These have resulted in the development of a number of methods for flood wave routing through reservoirs and down natural channels. Those methods believed most appropriate for use on Iceland rivers are discussed in subsequent sections of this appendix. Detailed procedures for their application are given.

D. ROUTING THROUGH RESERVOIRS

Flood Indication Method

The routing of a flood wave through a storage reservoir is the simplest application of routing procedures. In this case outflow and storage are interrelated and storage is controlled by the reservoir outlets, such as spillways, sluices, turbines, etc. so that it becomes a direct function of outflow. Equation (3) may be modified so as to simplify the routing of a flood wave through a reservoir constructed in a river channel. Let $t_2 - t_1$ in Equation (3) be equal to a small time interval designated as "t." Then

$$t\left(\frac{I_2 + I_1}{2} - \frac{O_2 + O_1}{2} \right) = S_2 - S_1 \quad \ldots\ldots\ldots\ldots \quad (4)$$

and

$$I_2 + I_1 - O_2 - O_1 = \frac{2S_2}{t} - \frac{2S_1}{t} \quad \ldots\ldots\ldots\ldots \quad (5)$$

Transpose all known quantities to the left side of the equation, then:

$$(I_2 + I_1) + \left(\frac{2S_1}{t} - O_1\right) = \frac{2S_2}{t} + O_2 \quad \ldots\ldots\ldots\ldots \quad (6)$$

The solution of this equation is known as the Flood Indication Method. In this solution all values of inflow, I_1 and I_2 as well as all values of outflow, O_1 and O_2 should be in cubic meters per second; all values of storage S_1 and S_2 should be in cubic meters; and time, t, should be in seconds. Given the inflow hydrograph to the reservoir; the physical characteristics of the reservoir; and the characteristics of all reservoir outlets, such as spillways, sluices, turbine discharges, etc., Equation (6) can be solved to give the outflow hydrograph at the dam and the time-stage relationship for the reservoir. The following steps are required in application of the Flood Indication Method for routing a flood wave through a long, deep reservoir:

a. Step 1 - Determination of Reservoir Capacity. Determine the capacity, or storage available in the reservoir from field survey data, or by planimetering suitable topographic maps of the reservoir area. Plot a curve to show the relation between elevation and storage below a flat pool. If the reservoir is a short, shallow one, backwater curves may be determined for its upstream portion, and storage between these and the level pool may also be considered in the routing process. Backwater storage curves can be determined and plotted for various flat pool elevations, using various values of inflow as parameters. These are known as Reservoir Storage Curves.

b. Step 2 - Selection of Time Interval, t. Select a convenient time interval, t, for use in the routing process. This interval should be short enough so that the position of the inflow and outflow hydrographs, as well as the portion of the reservoir capacity curve corresponding to t, can be assumed to be linear functions of t. The shorter the interval selected, the more accurate and satisfactory will be the results from the routing procedure. Therefore t may be taken as a day, a portion of a day, or a few hours, depending upon the shape of the inflow hydrograph and the reservoir storage curve.

c. Step 3 - Reservoir Outflow Curve. Calculate the outflow capacity of the spillway, the sluices, and the turbines, and all other outlets for various reservoir pool elevations. Plot outflow curves for each of these to show the relationship between reservoir level and their discharge.

Combine the values from all of these curves and plot a new curve to show the relationship between reservoir pool elevation and total outflow for that elevation. This is called a total reservoir outflow curve.

d. Step 4 - Storage Indication Curve. Select a number of intervals of reservoir pool elevation and tabulate their corresponding level storage in cubic meters, as read from the Reservoir Level Storage Curve determined in Step 1. For these same elevations tabulate the corresponding total discharge in cubic meters per second as read from the Reservoir Outflow Curve, determined in Step 3. From these data determine $\frac{2S}{t} + O$ for each selected elevation and tabulate. Plot a curve to show the relationship between O and $\frac{2S}{t} + O$. This is called a Storage Indication Curve, and represents the right-hand side of Equation (6). A typical tabulation for Step 4 is shown in Table 1 and a Storage Indication Curve on Figure 1.

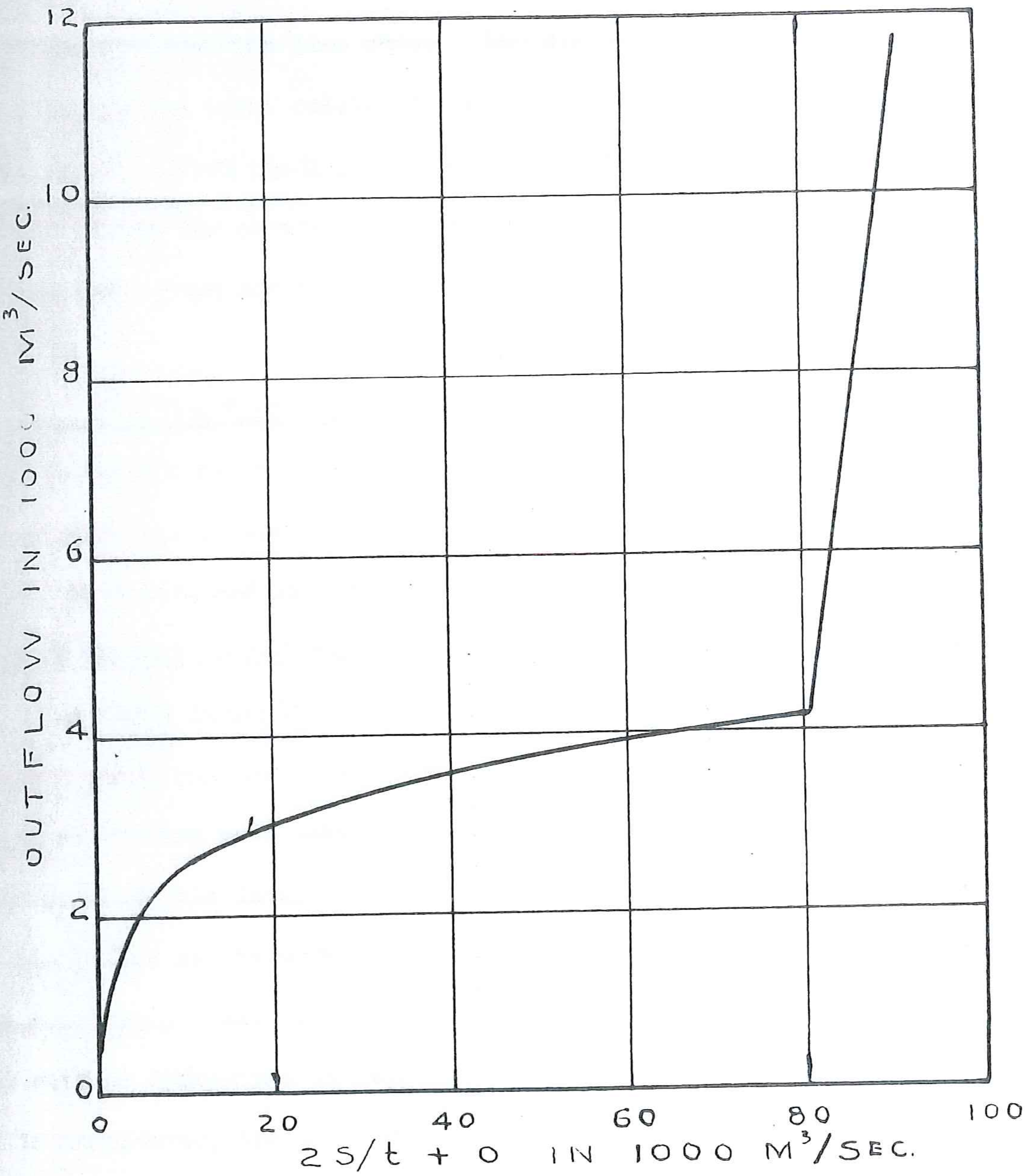
e. Step 5 - Reservoir Outflow Hydrograph. Tabulate the inflow hydrograph as shown in Table 2, and solve Equation (6) for each time interval, using the storage Indication Curve as shown in Figure 1. Table 2 shows a typical tabulation for solution of this problem. At the start of the solution the first value of O in Column (6) of Table 2 is known or is assumed. The first value in Column (5) is the value of $\frac{2S}{t} + O$ from the Storage Indication Curve of Step 4, that corresponds to the first value of O in Column (6). Column (7) in each case is the value in Column (5) minus twice the value in Column (6). In subsequent lines after the first line, the value in Column (5) is the value in Column (4) plus the value

TABLE 1

COMPUTATIONS FOR STORAGE INDICATION CURVE
(t = 7200 sec.)

Elevation Meters	Storage S M^3 X1000	$\frac{2S}{t}$ M^3/sec	Outflow O M^3/sec	$\frac{2S}{t} + O$ M^3/ sec
(1)	(2)	(3)	(4)	(5)
525	3	0.8	13.8	14.6
530	13	3.6	61.0	64.6
540	47	13.1	105.6	118.7
560	268	74.0	161.0	235.0
580	749	208.0	202.0	410.0
600	1545	429.0	236.0	665.0
620	2840	789.0	265.0	1054.0
640	4780	1328.0	293.0	1621.0
660	7360	2044.0	317.0	2361.0
680	10500	2917.0	340.0	3257.0
700	14110	3919.0	361.0	4280.0
720	18360	5100.0	381.0	5481.0
740	23100	6420.0	401.0	6820.0
750	25700	7140.0	409.0	7550.0
755	27000	7500.0	413.0	7910.0
758	27800	7720.0	697.0	8420.0
760	28400	7890.0	1022.0	8910.0

FIGURE 1.



STORAGE INDICATION CURVE

in Column (7) from the line above. The discrepancy in Table 2 between total inflow and total outflow is about two-tenths of one percent and probably resulted from reading values from the Storage Indication Curve. It is well within the accuracy of the original hydrologic data. The Reservoir Outflow Hydrograph may be plotted from the data in Columns (2) and (6) of Table 2.

f. Step 6 - Reservoir Fluctuations. Fluctuations of the reservoir pool elevations as shown in Column (8) of Table 2 were obtained by applying outflow values shown in Column (6) to the Total Reservoir Outflow Curve, as determined in Step 3.

Routing Through Short, Shallow Reservoirs

The Flood Indication Method may also be used for routing flood waves through short, shallow reservoirs. Technically speaking, routing through these reservoirs must take into account the effect of the additional storage above the level pool and below the backwater curves produced by the inflow at the upper portion of the reservoir. For steep streams and relatively small reservoirs, the effect of this additional storage on the outflow hydrograph is relatively small and is often neglected. When it is considered, the solution of Equation (6) by the Flood Indication Method can be accomplished only by "cut and try" procedures, since total reservoir storage is no longer a function of outflow only, but is also a function of inflow as well. Application of the Coefficient Method, described later for use on natural river channels, provides a better solution for small, shallow reservoirs when backwater storage is considered.

TABLE 2

ROUTING COMPUTATIONS FOR RESERVOIRS

Date	Time Hours	I M^3/sec	$I_1 + I_2$ M^3/sec	$\frac{2S}{t} + O$ M^3/sec	O M^3/sec	$\frac{2S}{t} - O$ M^3/sec	Pool Elevation Meters
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Mar 8	00	21		20	21	− 22	526
	02	60	81	59	60	− 61	530
	04	215	275	214	152	− 90	556
	06	345	560	470	211	+ 48	585
	08	380	725	773	247	+ 279	607
	10	430	810	1089	268	+ 553	622
	12	475	905	1458	287	+ 884	635
	14	525	1000	1884	302	+1280	647
	16	605	1130	2410	318	+1774	661
	18	665	1270	3044	336	+2372	676
	20	730	1395	3767	352	+3063	692
	22	780	1510	4573	368	+3837	706
	24	870	1650	5477	383	+4721	723
Mar 9	02	770	1640	6361	399	+5563	738
	04	700	1470	7033	402	+6229	742
	06	665	1365	7594	409	+6776	749
	08	630	1295	8071	494	+7083	756
	10	550	1180	8263	600	+7063	757
	12	490	1040	8103	525	+7053	756
	14	360	850	7903	413	+7077	755
	16	330	690	7767	410	+6947	751
	18	275	605	7552	409	+6734	750
	20	195	470	7204	402	+6400	742
	22	160	355	6755	400	+5955	739
	24	130	290	6245	396	+5453	735
Mar 10	02	105	235	5688	388	+4912	728
	04	88	193	5105	377	+4351	716
	06	54	152	4503	366	+3771	704
	08	47	111	3882	664	+3154	703
	10	30	77	3231	340	+2551	680
	12	21	51	2602	322	+1958	664
	14	20	41	1999	304	+1391	649
	16	20	40	1431	258-	+ 861	634
	18	19	39	900	255	+ 390	613
	20	19	38	428	205	+ 18	582
	22	19	38	560	62	− 68	530
	24	19	38	———	19	—————	526
Total		11827			11851		

E. ROUTING THROUGH NATURAL RIVER CHANNELS

Flood Routing Methods.

Routing of flood waves down natural river channels is fundamentally the same as routing them through reservoirs constructed in these channels. In general, each river is considered to be divided into sections, or reaches, and each reach is treated as a natural reservoir. However, storage in natural reaches depends upon both inflow and outflow and the solution of the routing problem is more complex than for reservoirs. Storage in natural river channels also depends upon the slope and other hydraulic characteristics of the reach and upon its physical control features.

Various investigators have developed a number of differential equations and specialized mathematical procedures for routing flood waves through natural river channels. Their application is too laborious for practical use and approximate methods of solution have been developed and are usually employed. Two of these methods, the Coefficient Method and the Working Value Method, are widely used and are described below.

Selection of River Sections, or Reaches

The river channel is divided into sections, or reaches, to simplify computations in whatever routing method is selected. The storage characteristics of a river should be carefully considered in the selection of routing reaches and the ends of these reaches should be chosen with these in mind. Major changes in river characteristics as well as channel controls, are generally indicated by abrupt changes in water surface profiles.

These abrupt changes should be considered in selecting the limits for routing reaches. The channel should be subdivided into reaches whose ends are located whenever possible at streamflow stations, so that recorded hydrographs may be used in determining the channel storage in such reaches. The lower end of a routing section should also be located at any channel control which creates a relatively large amount of channel storage in the pool-type reach above it. The upper end of this reach should be located at the upper end of the pool-type channel section. If a long reach of river occurs between two pool-type reaches, it should be subdivided into shorter reaches. Each of these should have a length such that the time of water travel through it, is approximately equivalent to the time interval Δt, selected for the routing operation. Whenever a major tributary enters the main stream the lower end of a routing reach should be located immediately below this junction. This is necessary to facilitate the routing of the inflow from the major tributary and to determine its affect on the flood wave moving down the main stream.

Coefficient Method

This method is based on the assumption, that the storage under any instantaneous profile through a reach of river, can be separated into prism storage and wedge storage and that the relationship between them can be approximated by means of coefficients. Early development and practical application of this method were made by Gerald T. McCarthy and others, in connection with studies of the Muskingum Conservancy District Flood Control Project made by the Corps of Engineers, United States Army in

1934 and 1935. It is often referred to, therefore, as the Muskingum Method in engineering literature. The principal difficulty in the application of this method is the determination of the required coefficients. One of its advantages is that it may be used, not only for routing hydrographs, but also for routing reductions in natural flows, resulting from operation of storage reservoirs. Since it expresses outflow as the summation of the products of inflow and routing coefficients, it produces a flood recession of the exponential form that retains the same recession coefficient as that of the inflow hydrograph.

The Coefficient Method is based upon the assumption that storage within a routing reach can be related to river discharges at each end of the reach by the following routing equation:

$$S = K\left[XI + (1-X)\,O\right] = KO + KX(I - O) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

In Equation (7), O and I represent simultaneous values of outflow and inflow, respectively, for the reach; S represents storage in the reach; and K and X are constants. K is the travel time through the reach of the center of mass of the flood wave hydrograph. X is the relative weight of the inflow upon the storage in the reach.

a. Prism Storage. The term KO in Equation (7) has been considered as representing prism storage. It represents the volume of water in the reach that is moving under a flow profile, where the inflow into the reach is equal to the outflow from the reach.

b. Wedge Storage. The term KX(I-O) in Equation (7) has been considered as representing wedge storage. It is the additional storage, moving on top of the prism storage, when either the inflow is greater than the outflow, or the outflow is greater than the inflow. In the first instance water is going into channel, or valley storage, and in the second it is being released from such storage. In each case, inflow, outflow, and storage are variables that are changing with respect to time, t.

c. Storage Curve. A storage curve is a curve showing the relationship between river discharge and channel storage. The most common type is one obtained by plotting outflow at the downstream end of a reach against channel, or valley storage, in the reach. Although the amount of water in channel storage is related to the stage and outflow at the downstream end of a routing reach, the relationship is not a fixed one. When the stream is rising the storage relation will be greater than that for a constant stage. When the stream is falling the storage relation will be less than that for a constant stage. The difference in storage is dependent upon the rate of rise, or fall, in the outflow and inflow of the reach. As a result of this effect, when storage is plotted against outflow, a loop curve usually results. If inflow is plotted against storage the loop curve is usually reversed. The Coefficient Method has been developed to determine the relative effect of both outflow and inflow up on channel storage and thus permit a single storage curve to be used for routing.

d. Determination of Storage Curve by Surveys. The storage curve, for each selected reach, may be determined by survey methods. This will require cross sections of the river to be determined from field measurements, or from topographic maps. The areas of these cross sections are computed for various river elevations at each point and a curve plotted to show the area-elevation at each cross section. Determine a similar curve for each cross section to show the relationship between hydraulic radius of the section and various elevations above the river bed. Apply the Manning Formula, and using values from these curves and a marked water surface profile from field surveys, determine the values at each cross section for Manning's roughness factor, N. Assume that the computed values of n, corresponding to the surveyed profile, are applicable to all flood wave stages at the respective cross sections. Again apply the Manning Formula and the computed values of n and compute natural channel flow profiles for various quantities of flow. The range in flow should be equal to that anticipated in the flood waves to be investigated. Use these flow profiles and the area-elevation curves to determine the volume under the profiles, between each pair of cross sections, for a series of flows. Assume this volume to be equal to the average of the areas of each pair of sections, multiplied by the length of channel between them. Sum up the various storage products for each routing reach and plot storage as abscissae and the respective flows for the profiles considered as ordinates. The resulting curve is the Storage Curve for each reach. Two factors make the above determination of the storage-discharge relationship by field surveys, usually a long procedure. First, most stream channels are irregular and numerous cross sections must be surveyed. Also, the areas and hydraulic radii of cross sections must be determined for successive stages corresponding to given discharges. Second, flow profiles must be computed throughout the channel length in order to compute the channel storage required to plot the Storage Curve.

This method gives a reasonably high order of accuracy and must be used where streamflow stations are not available to provide stage and discharge hydrographs at the ends of routing reaches.

e. Determination of Storage Curves from Hydrographs. An easier method of computing the storage curve is the use of known hydrographs of inflow and outflow to obtain the average inflow and outflow and determining storage by solving Equation (6) for changes in storage. Equation (6) shows that for each routing reach during a time interval, t, the average inflow minus the average outflow, is equal to the change in channel storage, ΔS. The inflow into each routing reach, however, must include all tributary inflow and all local inflow estimated for ungaged areas. The change in storage, ΔS, must be computed for successive time intervals beginning at the start of the flood wave passage at the upper end of the routing reach. The total storage at any time, during the flood wave passage, can be computed as the sum of these storage increments from the beginning of the wave passage. Use the same relatively low flow to determine the beginning and end of the storage computations. Then the storage at the end should approximately equal that at the beginning of the period, and the gain in storage during the rising side of the hydrograph should equal the release, or loss, of storage during the hydrograph recession. The net accumulation of storage at the end of the computation should be equal to zero. This provides a valuable check on storage computations. However, if the accumulated storages do not check, it is often due to errors in estimated inflow to the reach from ungaged areas, and an adjustment should be made.

f. Determination of X. The effect of inflow upon storage is indicated by X in Equation (7). The value of X will normally vary from 0.0 to 0.5. When X is equal to zero, the storage is a function of the outflow. This is the case for reservoirs and for rivers with very flat slopes. For steeper rivers with higher velocities, the value of X increases until in the extreme case it approaches 0.5 and the flood wave is translated, or moved, through the reach without any appreciable change in shape, or in peak rate of flow. To determine the value to use for X for each routing reach, assume values of X, ranging from 0.0 to 0.5, and for each value weight the inflow and outflow from known reach hydrographs by the formula of $XI + (1 - X)O$. Plot these weighted values as ordinates and their corresponding storage increment values, ΔS, as abscessae for each routing reach. The value of X, which reduces the loop in the Storage Curve to the smallest loop, or series of loops, so that it approaches a straight line, is the proper value of X to use in routing flood waves through the reach. Figure 2 shows several storage curves and the effect resulting from variation in X.

g. Determination of K. The term K in Equation (7) is a very important term and is very closely related to the travel time of the reach. The most direct method for determination of K is by comparison of actual hydrographs at the ends of a reach by inverse flood routing. In this method Equations (4) and (7) are solved for K to obtain the following equation:

$$K = \frac{0.5\Delta t\,[(I_2 + I_1) - (O_2 + O_1)]}{X(I_2 - I_1) + (1-X)(O_2 - O_1)} \quad \ldots\ldots\ldots\ldots\ (8)$$

This process has been described in a publication of the Engineer School, United States Army, Fort Belvoir, Virginia as follows:

> ".....successive values of the numerator and denominator (of Equation 8) are accumulated for floods for which the inflow and outflow are known with X as a parameter. The accumulated numerator values are plotted as abscissae and the accumulated denominator values as ordinates. The result is a series of curves for various values of X. The one approaching more nearly to a straight line for the entire flood satisfies most closely the equation and therefore determines the proper X for the reach. The K value is the reciprocal of the slope of the curve. Because the limits of accuracy of the runoff data usually available, it is preferable to compute K and X for several floods and to adopt for routing an average of these values."

Table 3 and Figure 2 illustrate this procedure, using material taken from the above source. In Table 3, Column 2 represents the inflow into the reach and includes inflow at the upper end of the reach, plus estimated local and tributary inflow entering between the two ends of the reach. This inflow has been adjusted so that its total volume is equal to the total volume of the outflow at the lower end of the reach. The numerators and denominators in Equation (8) were evaluated for each

TABLE 3

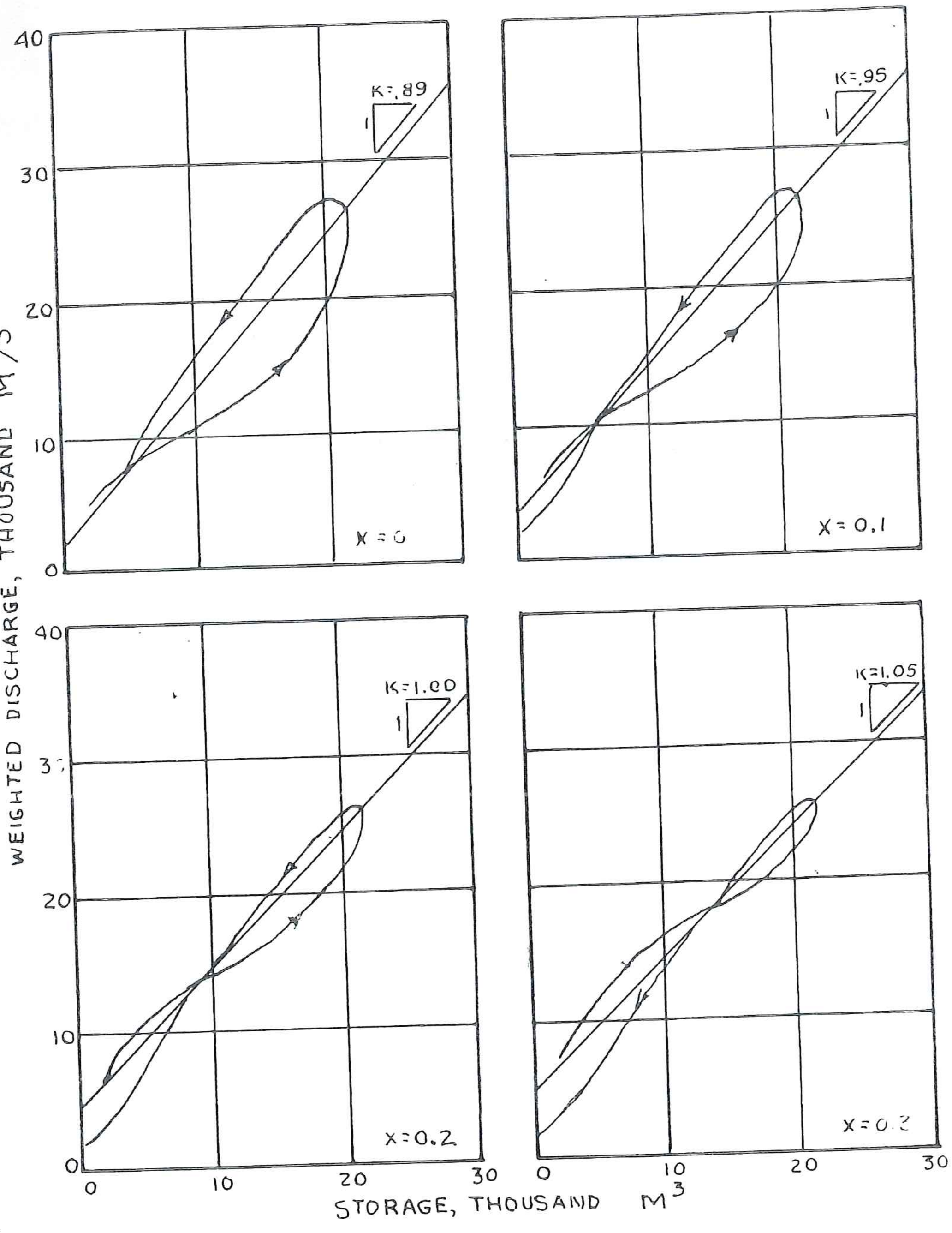
DETERMINATION OF K AND X — $K = \frac{\text{Numerator, } N}{\text{Denominator, } D} = \frac{0.5\Delta t\,(I_2+I_1) - (O_2+O_1)}{X(I_2-I_1) + (1-X)(O_2-O_1)}$

COEFFICIENT METHOD

TIME Δt=0.5 day	IN-FLOW 1/	OUT-FLOW 2/	I_2+I_1	O_2+O_1	I_2-I_1	O_2-O_1	N 3/	ΣN	VALUES OF D AND ΣD FOR ASSUMED VALUES OF X: X = 0, D 4/	X = 0, ΣD	X = 0.1, D 4/	X = 0.1, ΣD	X = 0.2, D 4/	X = 0.2, ΣD	X = 0.3, D 4/	X = 0.3, ΣD
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Feb 26 am	2.2	2.0	16.7	9.0	12.3	5.0	1.9		5.0		5.7		6.5		7.2	
pm	14.5	7.0	42.9	18.7	13.9	4.7	6.1	1.9	4.7	5.0	5.6	5.7	6.5	6.5	7.5	7.2
27 am	28.4	11.7	60.2	28.2	3.4	4.8	8.0	8.0	4.8	9.7	4.6	11.3	4.5	13.0	4.3	14.7
pm	31.8	16.5	61.5	40.5	-2.1	7.5	5.2	10.0	7.5	14.5	6.7	15.9	5.6	17.5	4.6	19.0
28 am	29.7	24.0	55.0	53.1	-4.1	5.1	0.5	21.2	5.1	22.0	4.1	22.6	3.2	23.1	2.3	23.6
pm	25.3	29.1	45.7	57.5	-4.9	-0.7	-2.9	21.7	-0.7	27.1	-1.1	26.7	-1.5	26.3	-2.0	25.9
Mar 1 am	20.4	28.4	36.7	52.2	-4.1	-4.6	-3.9	18.8	-4.6	26.4	-4.6	25.6	-4.5	24.8	-4.4	23.9
pm	16.3	23.8	28.9	43.2	-3.7	-4.4	-3.6	14.9	-4.4	21.8	-4.3	21.0	-4.3	20.3	-4.2	19.5
2 am	12.6	19.4	21.9	34.7	-3.3	-4.1	-3.2	11.3	-4.1	17.4	-4.0	16.7	-3.9	16.0	-3.9	15.3
pm	9.3	15.3	16.0	26.5	-2.6	-4.1	-2.6	8.1	-4.1	13.3	-4.0	12.7	-3.8	12.1	-3.6	11.4
3 am	6.7	11.2	11.7	19.4	-1.7	-3.0	-1.9	5.5	-3.0	9.2	-2.8	8.7	-2.8	8.3	-2.6	7.8
pm	5.0	8.2	9.1	14.6	-0.9	-1.8	-1.4	3.6	-1.8	6.2	-1.7	5.9	-1.6	5.5	-1.6	5.2
4 am	4.1	6.4	7.7	11.6	-0.5	-1.2	-1.0	2.2	-1.2	4.4	-1.2	4.2	-1.1	3.9	-0.9	3.6
pm	3.6	5.2	6.0	9.8	-1.2	-0.6	-1.0	1.2	-0.6	3.2	-0.6	3.0	-0.7	2.8	-0.8	2.7
5 am	2.4	4.6						0.2		2.6		2.4		2.1		1.9
pm																

1/ Inflow to the reach has been adjusted to equal the volume of the outflow
2/ Outflow at lower end of reach
3/ Numerator, N is ½ Δt (Column 4 - Column 5)
4/ Denominator, D is [Column 7 + X (Column 6 - Column 7)]

FIGURE 2



CHANNEL STORAGE CURVES

period using four assumed values of X.

Figure 2 shows the accumulated numerator, or storage values, in Column (9) of Table 3 plotted against the corresponding accumulated denominator or weighted outflow values as shown in Columns 11, 13, 15, and 17 of Table 3. The best fit is assumed to be that for which there is the least variation in the storage curve from a single line passing through it. On Figure 2, the fit appears to be satisfied best for X = 0.2 and K = 1 day, the mean line in this instance being taken as straight throughout the range of discharges. If, to conform to a criteria that $K = \Delta t = 0.5$ day, this reach was subdivided into two equal reaches, the value of K for each reach would be 0.5 day, assuming a constant wave movement throughout the two reaches. However, X would not necessarily retain the value of 0.2.

The ratio of reach length to K gives the rate of flood-wave movement. This rate can be applied to reaches of shorter or longer length unless there is a marked difference in storage characteristics between the desired reach and the reach for which K was derived.

h. Determination of Coefficients. Equations (4) and (7) may be combined to yield:

$$O_2 = C_1 I_2 + C_2 I_1 + C_3 O_1 \quad \ldots\ldots \quad (9)$$

Where the coefficients C_1, C_2, and C_3, have values in terms X, K and Δt as follows:

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X) + \Delta t} \quad \ldots\ldots \quad (10)$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X) + \Delta t} \quad \ldots\ldots \quad (11)$$

$$C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

The sum of the three coefficients must be equal to unity.

i. Coefficient Routing. Routing by the Coefficient Method is a fairly simple process after the storage curve, and values for X, K, C_1, C_2 and C_3 have been determined as explained above. This method is well suited to use in a computing machine with an accumulative multiplier. The computations used in this method of routing are illustrated in Table 4. First select the time interval, t, and then compute the values for X and K from the reach storage curve and from hydrographs for the reach. Use these values and compute values for coefficients C_1, C_2 and C_3. Substitute the values of these coefficients in Equation (9) and solve for O_2. Table 4 illustrates a method for tabulating these computations. Finally, plot the outflow values in Column 6 of Table 4 against their corresponding time as shown in Column (1) to give the outflow hydrograph at the lower end of the reach. This outflow hydrograph then becomes the inflow hydrograph at the upper end of the next downstream reach. The process is repeated for each of the succeeding downstream reaches.

Working Value Method

The term K, in Equation (7) for the Coefficient Method, was assumed to have a constant value for all stages. In some rivers and under some conditions, K varies with the rate of change in outflow. On these rivers the Storage Curve cannot be averaged by a straight line and a different routing method must be employed. The Working Value Method has been devised for use with a variable value for K.

TABLE 4

ROUTING BY COEFFICIENT METHOD

t = 0.5 day   K = 0.5 day   X = 0.3

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X) + \Delta t} = 1/6 \quad C_2 = \frac{\Delta t + 2KX}{2K(1-X) + \Delta t} = 2/3 \quad C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t} = 1/6$$

$$O_2 = C_1 I_2 + C_2 I_1 + C_3 O_1$$

Time	Inflow	C_1I_2	C_2I_1	C_3O_1	Outflow
		$1/6\ I_2$	$2/3\ I_1$	$1/6\ O_1$	
(1)	(2)	(3)	(4)	(5)	(6)
	2.0				2.0
Feb 26 a.m.	2.0	0.33	1.33	0.33	1.99
p.m.	7.0	1.17	1.33	0.33	2.83
27 a.m.	11.7	1.95	4.67	0.47	7.09
p.m.	16.5	2.75	7.80	1.18	11.73
28 a.m.	24.0	4.00	11.00	1.96	16.96
p.m.	29.1	4.85	16.00	2.82	23.67
Mar 1 a.m.	28.4	4.73	19.40	3.94	28.07
p.m.	23.8	3.97	18.93	4.68	27.58
2 a.m.	19.4	3.23	15.87	4.59	23.69
p.m.	15.3	2.55	12.93	3.95	19.43
3 a.m.	11.2	1.87	10.20	3.24	15.31
p.m.	8.2	1.37	7.46	2.55	11.38
4 a.m.	6.4	1.07	5.47	1.89	8.43
p.m.	5.2	0.87	4.27	1.40	6.54
	208.2				208.70

The Working Value Method is based upon the assumption that:

$$D = XI + (1-X)O \quad \ldots \ldots \ldots \ldots \ldots \quad (13)$$

In this case the "working discharge," D, represents a hypothetical steady flow, that would result in storage in a given reach, equal to that that would be produced by given values of actual unsteady inflow, I, and outflow, O. If Equation (13) is solved for O, and this term is substituted in Equation (4), that equation may be rewritten as:

$$0.5\Delta t(I_2 + I_1) + S_1(1-X) - 0.5D_1\Delta t = S_2(1-X) + 0.5D_2\Delta t \quad . \quad (14)$$

If $S(1-X) + 0.5D\Delta t$ is called R, Equation (14) becomes:

$$R_1 + 0.5(I_2 + I_1)\Delta t - D_1\Delta t = R_2 \quad \ldots \ldots \ldots \ldots \quad (15)$$

The term R in Equation (15) is called a "working value" and becomes an index of storage in each reach.

<u>a. Development of R-D Curves.</u> The routing of a hydrograph by the Working Value Method requires the prior determination of the Working Discharge, D, and the Working Value, R, for each reach. A working curve, showing the relationship between D and R may be determined from the inflow and outflow hydrographs for each reach. The working values for D are computed from application of Equation (13). Working values for R are obtained from:

$$R = S(1-X) + 0.5D\Delta t \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (16)$$

A Working Value R - D Curve is obtained by plotting values of D as ordinates and the corresponding values of R as abscissae. If the use of actual hydrographs, results in a scattering of the R-D points, draw a smooth curve by eye to represent the mean of the points. In some cases this procedure may result in a large variation in R - D curves developed for different flood wave events. This will indicate that the initial assumptions for reach lengths, and for the values of X require revision. A wrong assumption for the value of X will result in a large loop or series of loops in the rough R - D Curves. Too long a reach and too great a time interval, t, will also be reflected in the R - D Curves and these may also require revision. Trial computations, using different values of X and t should be made until the R - D Curves approach straight lines.

Working Value Routing Procedure

Routing by use of the Working Value Method is accomplished by steps similar to those used in the Flood Indication Method for Reservoirs. The method used for tabulation of the results of these steps is shown in Table 5.

a. Step 1 - Determination of K. Select inflow and outflow hydrographs for a flood wave passing through each reach considered. By inspection of these hydrographs make an estimate of a value for K. If K is not readily apparent by inspection, compute the time of travel of the center of mass of the flood wave hydrographs and use this value for K.

b. Step 2 - Selection of Time Interval, Δt. Select a time interval between the values for t = K and t = 2KX and use for Δt. Inspect the flood wave hydrographs to insure that at least 5 or 6 intervals will occur before the peak of the hydrographs are reached.

c. Step 3 - Determination of X. Plot storage curves, using values of X ranging from 0.0 to 0.5, and select the value of X to the nearest tenth that produces the smallest loop, or loops, in the Storage Curve. Use this value in further routing computations.

TABLE 5

ROUTING BY WORKING VALUE METHOD

$t = 1$ day, $X = 0.2$

$$O = D - \frac{X}{1-X}(I-D)$$

Time	Inflow I	Aver. I	Working Discharge D	Col. 3 Minus Col. 4	Working Value of R	I Minus D	$\frac{X}{1-X}(I-D)$	Outflow
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
May 1	45		31		26.0	14.0	3.5	27.5
		50		19				
2	55		43.8		45.0	11.2	2.8	41.0
		60		16.2				
3	65		53.2		61.2	11.8	2.9	50.3
		75		21.8				
4	85		65.9		83.0	19.1	4.8	61.1

Explanation

Column 1 - Time of inflow intervals.

Column 2 - Tabulation of inflow hydrograph

Column 3 - Average of two inflows — $\frac{I_1 + I_2}{2}\Delta t$

Column 4 - Working discharge D equal to XI + (1-X)O, for the first period and selected from R - D curve for succeeding periods for corresponding values of R_2

Column 5 - Column 3 minus Column 4

Column 6 - Working value of R taken from R - D curve at beginning of rise (26.0) and $R_2 = R_1 + 0.5(I_1 + I_2)\Delta t - D_1$ t for succeeding periods (Col. 5 plus previous R)

Column 7 - Column 2 minus Column 4

Column 8 - Column 7 times $\frac{X}{1-X}$

Column 9 - The outflow hydrograph, Column 4 minus Column 8.

d. Step 4 - Determination of R and D Values. Apply Equation (13) to determine values of D and Equation (15) to compute the corresponding values of R. Plot the Working Value Curve from these values. If additional flood wave hydrographs are available at both ends of the routing reach, make similar computations from them and compare the R - D Curves determined. Where a serious discrepancy occurs between these curves, revise the values of t and X selected to secure reasonably close agreement between such curves.

e. Step 5 - Determination of Outflow Hydrograph. Compute the outflow hydrograph from the inflow hydrograph for each reach, using Equation (14) and the R - D Working Value curve for the reach. To simplify its application Equation (14) may be rewritten in the following form:

$$O = D - \frac{X}{1-X}(I - D) \qquad (17)$$

The computations used in this routing method are illustrated in Table 5 and in the explanation accompanying it.

F. ROUTING AT A MAJOR STREAM JUNCTION

Major tributaries often enter the main stream and their inflow affects the progression of a flood wave down the main stream. Two general cases, relating to tributary effects, occur. The first case is where a tributary has a steep slope, or its flow is relatively small compared to the flows in the main stream. In this case tributary stages and flows are relatively unaffected by stages and flows in the main stream. The second case occurs when the slopes of the tributary and main stream are relatively flat and the tributary flow is large, as compared to that in the main stream. In this case the flow in both the tributary and the main stream mutually affect each other. Methods for handling each of these cases follow.

Case 1 - Minor Effect of Main Stream

Tributary stream flow introduces an independent variable in this case and special routing procedure must be used to solve this problem. The stages, and therefore the storage, in the routing reach on the main stream, depend upon the tributary inflow as well as upon the flood wave in the main stream. The Working Value Method is best suited for solving this situation, and routing reaches on the main stream should be selected so that the lower end of a reach is located immediately below the junction of the tributary.

a. Step 1 - Development of R - D Curves. Construct working R - D Curves for the main stream, using various values of tributary flows as parameters. A typical example of these curves is shown on Figure 9. Table 6 illustrates the method of arranging the routing computations.

TABLE 6

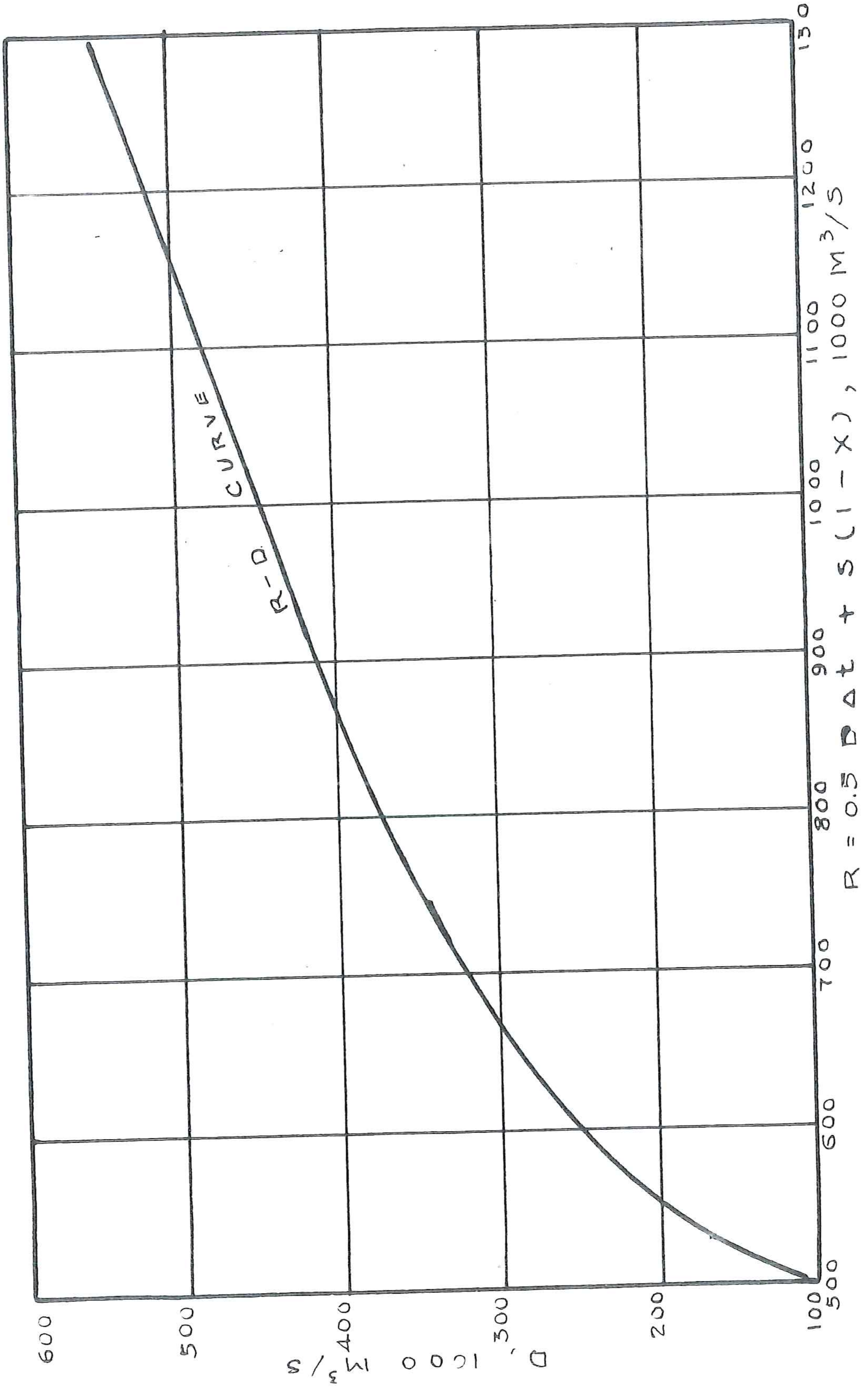
ROUTING BY WORKING VALUE METHOD-
JUNCTION OF MAJOR TRIBUTARY - CASE 1.
(Tributary flow conditions independent of main stream)

$t = 1$ day, $X - 0.2$ $\quad O = D - \frac{X}{I-X}(I-D)$

Time Days	Inflow I	Average Inflow $\frac{I_1+I_2}{2}$	Main Stream R	Main Stream D	Main Stream O	Tributary Discharge	Total Outflow
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	1050*		2485	1000*	988	226*	1214
		107.5					
2	1100*		2560	1010	988	262*	1250
		112.5					
3	1150*		2675	1042	1015	298*	1313
		117.5					
4	1200*		2800	1077	1046	329*	1375
		122.5					
5	1250*		2956	1110	1075	379*	1454

* Known Value

FIGURE 3


R-D CURVE
R = 0.5 D Δt + S (1 - X), 1000 M³/S
500
600
700
800
900
1000
1100
1200
1300
D, 1000 M³/S
100
200
300
400
500
600

b. Step 2. Tabulate known values of the main stream inflow hydrograph in Column 2 of Table 6. Tabulate similar known values of the hydrograph of the tributary at its mouth in Column 7 of Table 6.

c. Step 3. Enter the R - D Working Curve on Figure 2 with the tributary flow of 226 and an assumed working discharge, D, of 1000 for the main stream, to obtain a working value, R of 2485 for the first day. Enter this value in Column 4 of Table 6.

d. Step 4. Compute R_2 for the second day from:

$$R_2 = R_1 + 0.5(I_1 + I_2)\Delta t - D\Delta t = 2485 - 1075 - 1000 = 2560$$

Enter this value as the second value in Column 4.

e. Step 5. Enter the Working Value curves with the tributary discharge of 262 for the second day and the working value R of 2560, to obtain the working discharge, D, of 1010 for the main stream. Enter this value as the second item in Column 5.

f. Step 6. Compute the main stream outflow, O from:

$$O = D - \frac{X}{1-X}(I-D) = 1010 - \frac{0.2(1100 - 1010)}{0.8} = 988$$

Enter this value as the main stream outflow on second day, in Column 6.

g. Step 7. Determine total outflow at the downstream end of the routing reach by adding main stream outflow in Column 6 and tributary discharge in Column 7. 988 + 262 = 1250 as the outflow on the second day. Enter this value as the second item in Column 8.

h. Step 8. Repeat Steps 3 to 7 to obtain values for total outflow for each of the following days. Enter values for each day as shown in Table 6.

i. Step 9. Plot the outflow hydrograph from values in Column 8 of Table 6.

4

Case 2 - Major Effect of Main Stream

Flow in the main stream, and flow in a major tributary mutually affect each other when both streams have a relatively flat slope. This also may occur when the flow in the tributary is large in proportion to the flow in the main stream. No completely adequate routing procedure has been developed to solve this case. However, a trial and error method has been developed which gives reasonably satisfactory results. This procedure, however, may require several trials for each routing interval, before values of outflow are determined that will satisfy all the requirements.

a. Step 1 - Working Curves. Prepare two sets of Working Value Curves by the methods described above. One set is prepared for the main stream, using tributary flows as parameters. The second set is prepared for tributary flows, using main stream flows as parameters. Figure 4 illustrates these curves and Table 7 illustrates the computations required.

b. Step 2. Determine initial values of D and R for both the main stream and the tributary. The value of D is computed from:

$$D_1 = O_1(1 - X) + I_2X$$

The values of X were previously determined for the example in Table 7 as 0.15 for the tributary and 0.20 for the main stream. Then:

$$D_t = 226(0.85) + 250(0.15) = 192 + 38 = 230$$

$$D_s = 988(0.80) + 1050(0.20) = 790 + 210 = 1000$$

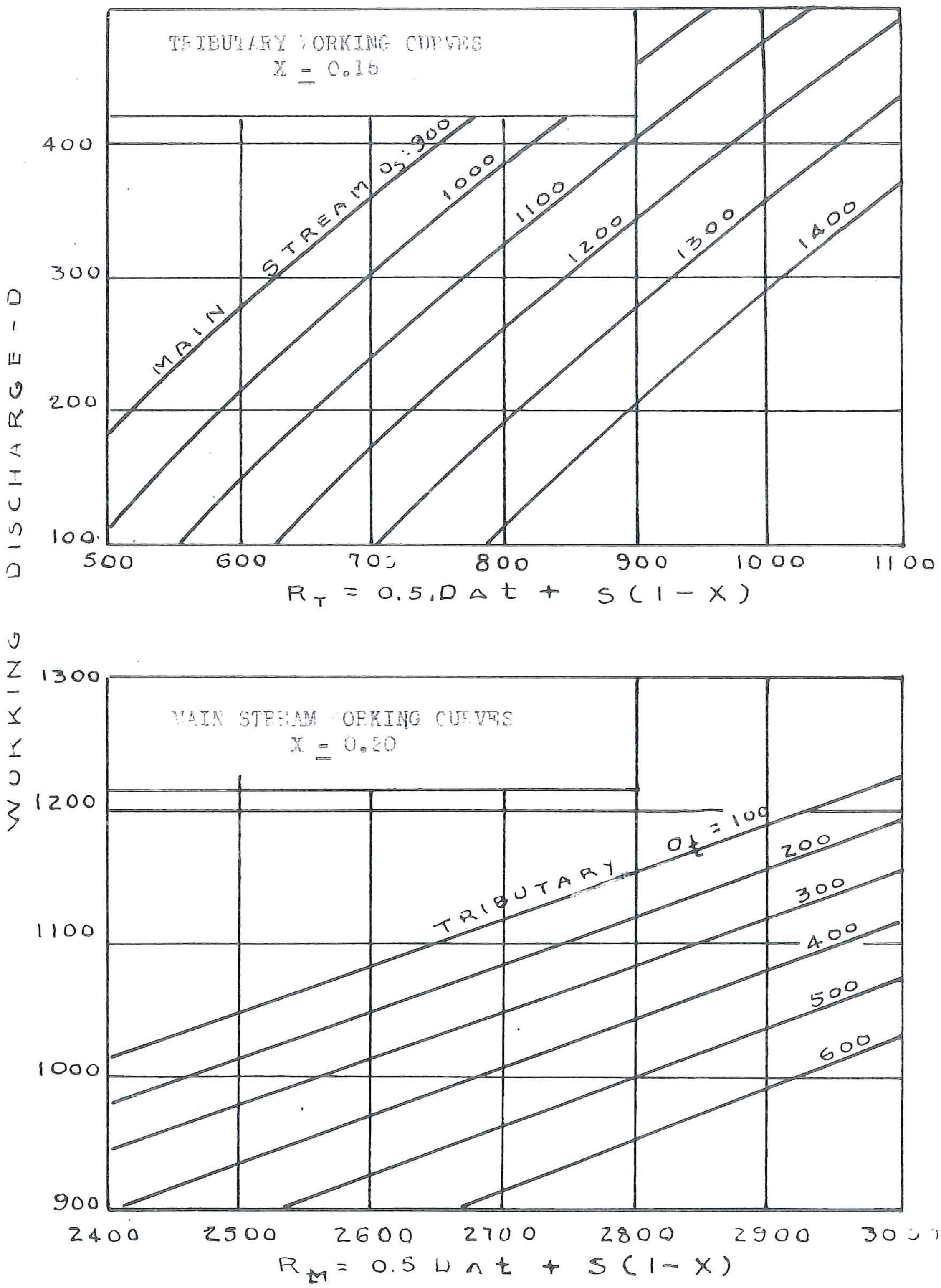
These values of D_t and D_s are entered as the initial values in Columns (5) and (10) of Table 7. The initial values of R for the initial value

TABLE 7
ROUTING WITH WORKING VALUE METHOD–
JUNCTION OF MAJOR TRIBUTARY – Case 2.
(Change of Discharge in Either Stream Affects Discharge in Other)

t = 1 day

	Tributary (X = 0.15)					Main Stream (X = 0.20)					
Time	I_t	$\frac{I_1 + I_2}{2}$	R_t	D_t	O_t	I_s	$\frac{I_1 + I_2}{2}$	R_s	D_s	O_s	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	250		606	230	226	1050		2485	1000	988	1214
2	300	275	651	270	265	1100	1075	2560			
					260				1012	990	
				268	262				1010	988	1250
3	350	325	708	320	315	1150	1125	2675			
					300				1040	1012	
				305	297				1042	1015	
				306	298				1042	1015	1313
4	400	375	777	360	353	1200	1175	2808			
					340				1073	1041	
				344	336				1078	1048	
				340	329				1077	1046	1375
5	450	425	862	406	398	1250	1225	2956			
					370				1115	1079	
				390	379				1110	1075	1454

FIGURE 4


TRIBUTARY WORKING CURVES
X = 0.15
MAIN STREAM Q_s = 900
1000
1100
1200
1300
1400
400
300
200
100
500
600
700
800
900
1000
1100
$R_T = 0.5 D \Delta t + S(1-X)$
WORKING DISCHARGE - D
MAIN STREAM WORKING CURVES
X = 0.20
TRIBUTARY Q_t = 100
200
300
400
500
600
1300
1200
1100
1000
900
2400
2500
2600
2700
2800
2900
3000
$R_M = 0.5 D \Delta t + S(1-X)$

are taken from the R - D curves determined in Step 1 for the known values of D and O for both the tributary and the main stream and are:

$$R_t(D_t = 230, O_t = 988) = 606$$

$$R_s(D_s = 2485, O_s = 226) = 2485$$

c. Step 3. Compute the value of R_2 from:

$$R_2 = R_1 + 0.5(I_1 + I_2)\Delta t = D_1 \Delta t$$

Then:

$$R_t = 606 + 275 - 230 = 651$$

$$R_s = 2485 + 1075 - 1000 = 2560$$

Enter these values in Columns 4 and 9 to Table 7 for the second day. Use these values to obtain trial values for D and O.

d. Step 4. Use the Tributary Working Curve and obtain trial values for D_t at end of first day, assuming that no change occurs in O_s during the interval.

$$D_t = (R_t = 651, O_s = 988) = 270 \text{ from curves.}$$

e. Step 5. Compute a trial value of O_t from:

$$O = D - \frac{X}{1-X}(I - D)$$

$$O_t = 270 - 0.177(300-270) = 265$$

This value is too high, so it is apparent that the true value of O_t will be between the trial value and the value at the beginning of Step 5. Select an intermediate value of 260 and determine new trial values of D_t and O_t.

f. Step 6. Use the Main Stream Working Curve and obtain a trial value of D_s.

$$D_s(R_s = 2560, O_t = 260) = 1012 \text{ from curves.}$$

g. Step 7. Compute a trial value for O_s from:

$$O = D - \frac{X}{I-X}(I - D)$$

$$O_s = 1012 - 0.25(1100 - 1012) = 1012 - 22 = 990$$

h. Step 8. Make a second approximation of D_t, using $R_2 = 651$ from Step 3, and $O_s = 990$ from Step 7. The Tributary Working Curves give $D_t = 268$.

i. Step 9. Compute a new trial value of O_t from using D_t of 268 from Step 8.

$$O = D - \frac{X}{I-X}(I-D)$$

$$O_t = 268 - 0.177(300 - 268) = 268 - 6 = 262.$$

j. Step 10. Make a second approximation of D_s using $R_t = 2560$ from Step 3, and $O_t = 262$ from Step 9. The Main Stream Working Curves give $D_s = 1010$.

k. Step 11. Compute O_s from $O = D - \frac{X}{I-X}(I-D)$ using $D_s = 1010$ from Step 10.

$$O_s = 1010 - 0.25(1100-1010) = 1010 - 22 = 988.$$

l. Step 12. Return to the Tributary Working Curves with the computed value of $R_t = 651$ from Step 3 and the second approximation of $D_s = 988$ from Step 11. These curves give a value of $D_t = 268$, which is the same as that determined from the second approximation in Step 8. Therefore, no further trials are necessary as further approximations will result in no further change in O_t or O_s.

m. Step 13. Continue the above trial and error process to determine O_t and O_s for each of the remaining days in the routing period.

n. Step 14. Plot the outflow hydrograph for the routing reach, from the values entered in Column 12 of Table 7 and determined by the above procedure.